Graviton Absorption by Non-BPS Branes

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Abstract

We consider the behaviour of neutral non-BPS branes probed by scalars and gravitons. We show that the naked singularity of the non-BPS branes is a *repulson* absorbing no incoming radiation. The naked singularity is surrounded by an infinite potential well breaking the unitarity of the scattering S-matrix. We compute the absorption cross section which is infrared divergent. In particular this confirms that gravity does not decouple for the non-BPS branes.

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1   Introduction

Non-BPS branes have been extensively studied recently both from the theoretical and phenomenological points of view[1]. In string theories the non-BPS branes appear to be unstable configuration in dimensions at odds with the usual BPS branes, i.e. $2p$-branes in type IIB strings and $(2p+1)$-branes in type IIA strings. One of the salient features of this construction based upon orbifolding coincident brane-antibrane pairs is the presence of a real tachyonic field with a non-trivial potential[2]. In particular the fate of the non-BPS branes has been linked to the rolling of the tachyon along its decreasing potential and its eventual condensation at the minimum of the potential. The precise description of the potential has been tackled using string field theory methods[3]. Recent advances in describing the decay of the unstable branes have appeared from the point of view of matrix theory and non-commutative geometry[4, 5, 6, 7]. Extensions to unstable M-branes have also been discussed[8].

Another approach to the issue of non-BPS branes comes from the supergravity side where exact solutions have been obtained and analysed[9, 10]. They describe charged and neutral configurations. The parameter space of Poincaré invariant solutions comprises three directions readily identified with the ADM mass, the charge and an extra integration constant c_1 which appears to be a function of the tachyonic vev. The decay of the unstable branes can be seen as a path in the parameter space[10].

An intriguing feature of these solutions is the presence of a naked singularitywith an attractive potential. It is then highly relevant to discuss the possible resolutions of this singularity before the eventual decay of the unstable non-BPS branes. One can test the nature of naked singularities by probing the supergravity solution with scalars and gravitons. In particular the quantum behaviour of gravitons is crucial. In the following we shall apply the wave-regularity criterion stating that naked singularities with a well-defined time evolution operator are quantum mechanically sensible[11, 12].

In a first part we will recall the structure of the neutral non-BPS solutions and show that the naked singularity is a *repulson*[13], a singularity reflecting perfectly any incoming radiation. Moreover the wave-regularity criterion is satisfied. Nevertheless we show that the potential well around the singularity traps wave-packets preventing the unitarity of the scattering S-matrix. In a second part we compute the absorption by this potential well and show that it possesses an infrared singularity obstructing the decoupling of gravity for non-BPS branes.

2   Scalar Absorption by Non-BPS Branes

2.1 The Scalar Potential

In the following we shall be interested in the scalar absorption by extended objects[14]. Let us consider the Klein-Gordon equation for a massless scalar field in a gravitational background corresponding to p-branes in ten dimensions

$$ds^2_{10} = e^{2A(\rho)}dx^2_{//} + e^{2B(\rho)}d\rho^2 + e^{2C(\rho)}\rho^2 d\Omega^2_{8-p} \tag{1}$$

with Poincaré invariance on the brane and rotational invariance in the transverse direction. We are considering the scattering of scalars governed by the equations

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu\Phi) = 0. \tag{2}$$

Let us now study the partial waves

$$\Phi = e^{-i\omega t}P_l(\Omega)\phi_l(\rho) \tag{3}$$

where P_l is an eigenfunction of the spherical Laplacian. It is easy to see that the wave equation can be cast in the form of a Schrodinger equation by putting

$$\phi_l(\rho) = (g^{\rho\rho}\sqrt{g})^{-1/2}F_l(\rho) \tag{4}$$

yielding

$$(\frac{d^2}{d\rho^2} - V(\rho))F_l = 0 \tag{5}$$

where the potential reads

$$V = -w^2 e^{2B(\rho)-2A(\rho)} + \frac{1}{\sqrt{U}}\frac{d^2\sqrt{U}}{d\rho^2} + \frac{l(l+7-p)}{\rho^2}e^{2B(\rho)-2C(\rho)} \tag{6}$$

and prime stands for $d/d\rho$. The potential depends on the form factor

$$U = g^{\rho\rho}\sqrt{g}. \tag{7}$$

The Klein-Gordon equation is covariant under changes of coordinates leading to the transformations of the potential and the wave function

$$\begin{aligned}
\tilde{F}_l(\rho^*) &= (\frac{d\rho^*}{d\rho})^{1/2}F_l(\rho) \\
\tilde{V} &= (\frac{d\rho^*}{d\rho})^{-2}V - \frac{1}{2}\{\rho,\rho^*\}
\end{aligned} \tag{8}$$

where the Schwarzian derivative is given by

$$\{\rho,\rho^*\} = (\frac{\rho''}{\rho'})' - \frac{1}{2}(\frac{\rho''}{\rho'})^2. \tag{9}$$

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We will mainly use the tortoise coordinates[14, 15]

$$d\rho^* = e^{B(\rho)-A(\rho)}d\rho \tag{10}$$

leading to the s-wave potential

$$V_{tort.} = -\omega^2 + \frac{1}{\sqrt{U}}\frac{d^2\sqrt{U}}{d\rho^{*2}}. \tag{11}$$

The Schrodinger equation can be written in a factorized form[16]

$$(\bar{Q}Q - \omega^2)\tilde{F}_0 = 0 \tag{12}$$

where Q and $\bar{Q}$ are formal adjoints for functions vanishing to second order at the origin

$$Q = -\frac{d}{d\rho^*} + \frac{1}{2}\frac{d\ln U}{d\rho^*}, \ \bar{Q} = \frac{d}{d\rho^*} + \frac{1}{2}\frac{d\ln U}{d\rho^*}. \tag{13}$$

The Hamiltonien $H = \bar{Q}Q$ is a symmetric operator $(g, Hf) = (Hg, f)$ where

$$(f, g) = \int d\rho^* f^*(\rho^*)g(\rho^*) + \int d\rho^* D_{\rho^*} f^*(\rho^*)D_{\rho^*}g(\rho^*) \tag{14}$$

for function [3]depending only on ρ^* provided that, at least, the no-flux boundary condition is satisfied

$$f^*(\rho^*)\frac{dg(\rho^*)}{d\rho^*}|_{\rho^*=0} = g^*(\rho^*)\frac{df(\rho^*)}{d\rho^*}|_{\rho^*=0}. \tag{15}$$

This is always satisfied if one restricts the domain $D(H)$ of H to the infinitely differentiable functions with compact support. With this choice the Hamiltonian is symmetric but is not guaranteed to be a self-adjoint operator.

On the other hand when the flux at the origin is non-zero then unitarity cannot be respected resulting in a non-zero absorption cross-section. Notice too that the Hamiltonian is a positive operator with two zero modes

$$\psi_1(\rho^*) = \sqrt{U}, \ \psi_2(\rho^*) = \sqrt{U}\int^{\rho^*}\frac{du}{U(u)}. \tag{16}$$

The rest of the spectrum is positive preventing the existence of tachyons.

In general it is impossible to find analytic expressions for the wave functions over the whole space. One has to resort to the matching technique relating the wave functions in two different patches. The tortoise coordinates are particularly suited close to the origin and at infinity.

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[3]The choice of the norm is crucial for the physical description of the singularities. Following [12] we choose a Sobolev norm as it is related to the energy of the scalar field ϕ. In particular fields with finite norm have finite energy.

2.2 Absorption by Non-BPS Branes

Let us give the metric in the Einstein frame. It is convenient to use isotropic coordinates[10]. The metric reads

$$ds_{10}^2 = \left(\frac{f_-}{f_+}\right)^\alpha dx_{//}^2 + f_-^{\beta_-} f_+^{\beta_+} (r^2 d\Omega_{8-p}^2 + dr^2) \tag{17}$$

where $f_\pm = 1 \pm r_0^{7-p}/r^{7-p}$. The exponents depend on the tachyon vev c_1 as

$$
\begin{aligned}
\alpha &= (7-p)\left(\frac{-|3-p|c_1 + 4k}{32}\right) \\
\beta_\pm &= \frac{2}{7-p} \pm (p+1)\left(\frac{-|3-p|c_1 + 4k}{32}\right) \\
k &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16}c_1^2}
\end{aligned}
\tag{18}
$$

where $\alpha \geq 0$ to guarantee the positivity of the ADM mass. It is convenient to use the near singularity coordinate $\rho^{7-p} = r^{7-p} - r_0^{7-p}$ to describe the space-time for $r \geq r_0$ away from the naked singularity at $r = r_0$. In this system of coordinates the metric reads

$$ds_{10}^2 = \tilde{f}_{++}^{-\alpha} dx_{//}^2 + \tilde{f}_+^{2(p-8)/(7-p)} \tilde{f}_{++}^{\beta_+} (d\rho^2 + \rho^2 \tilde{f}_+^2 d\Omega_{8-p}^2) \tag{19}$$

where $\tilde{f}_+ = 1 + r_0^{7-p}/\rho^{7-p}$ and $\tilde{f}_{++} = 1 + 2r_0^{7-p}/\rho^{7-p}$. The tortoise coordinates are given by

$$d\rho^* = \tilde{f}_+^{(p-8)/(7-p)} \tilde{f}_{++}^{(\alpha+\beta_+)/2} d\rho. \tag{20}$$

The metric becomes

$$ds_{10}^2 = \tilde{f}_{++}^{-\alpha}(dx_{//}^2 + d\rho^{*2}) + \tilde{f}_+^{-2/(7-p)} \tilde{f}_{++}^{\beta_+} \rho^2 d\Omega_{8-p}^2 \tag{21}$$

leading to

$$U = \tilde{f}_+^{(p-8)/(7-p)} \tilde{f}_{++}^{(\alpha+\beta_+)/2+1} \rho^{8-p}. \tag{22}$$

Close to the singularity it is convenient to study the behaviour of U and ρ^* as a function of ρ

$$
\begin{aligned}
U(\rho) &\sim S\rho^{-a} \\
\frac{d\rho^*}{d\rho} &\sim T\rho^{-1-a}
\end{aligned}
\tag{23}
$$

where the constants are defined by

$$a = p - 9 + (7-p)\frac{\alpha + \beta_+}{2} \tag{24}$$

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and $S = 2^{-p\alpha/2+(8-p)\beta_+/2} r_0^{p-8-(7-p)(p\alpha-(8-p)\beta_+)/2}$, $T = 2^{(\alpha+\beta_+)/2} r_0^{p-8+(7-p)(\alpha+\beta_+)/2}$.
Notice that $a < 0$ implying that ρ^* vanishes for vanishing ρ. This implies that the naked singularity at the origin is time-like. The resulting Schrodinger equation is of the Bessel type of zeroth order with generalized eigenvalues

$$\tilde{\psi}_\omega^1(\rho^*) = \sqrt{\rho^*} J_0(|\omega|\rho^*), \ \tilde{\psi}_\omega^2 = \sqrt{\rho^*} N_0(|\omega|\rho^*). \tag{25}$$

Close to the singularity the constant term in the potential is negligible implying that all the solutions behave like the two zero modes

$$\psi_1(\rho^*) = \sqrt{\rho^*}, \ \psi_2(\rho^*) = \sqrt{\rho^*} \ln \rho^*. \tag{26}$$

It is easy to see that no flux reaches the singularity and the s-wave absorption cross section by the singularity vanishes exactly. The same result holds for the higher partial waves.

2.3 Naked Singularities

The previous result sheds some light on the nature of the naked singularity at the origin. Classically the naked singularity is attractive as can be seen from the form of the potential

$$V_p(\rho^*) = -\omega^2 - \frac{1}{4\rho^{*2}} \tag{27}$$

close to the origin. Quantum mechanically the potential is so steep that the scalar probes are totally reflected. The singularity is then a *repulson*[13]. One can even go deeper in the analysis of the singularity by describing the time evolution of scalar probes in the vicinity of the singularity[11, 12]. To do that let us write the massless Klein-Gordon equation in the form

$$\frac{\partial^2 \phi}{\partial t^2} = -M\phi \tag{28}$$

where M is a second order partial differential operator depending only on the spatial derivatives. After a change of variable, M reduces to the Hamiltonian H. The Klein-Gordon equation defines a unique time evolution provided it can be written in the Schrodinger form

$$\frac{\partial \phi}{\partial t} = iM^{1/2}\phi \tag{29}$$

for a unique self-adjoint operator $M^{1/2}$. This is equivalent to finding a unique self-adjoint extension to the symmetric operator H, i.e. the Hamiltonian H is essentially self-adjoint.

There is a useful criterion of essential self-adjointness. Let us consider the eigenvalue problem

$$\nabla^\mu \nabla_\mu \phi = \pm i\phi \qquad (30)$$

in the non-BPS background. It reduces to a Schrodinger problem in a complex potential

$$V = V_p \pm i g_{\rho\rho}. \qquad (31)$$

Denote by $n_\pm$ the number of normalizable solutions to (30). As the operator H is real one has $n_+ = n_-$ implying that there always exists self-adjoint extensions. Now the operator is essentially self-adjoint provided $n_\pm = 0$, i.e. the solutions are not normalizable. This is achieved provided that one of the solutions for each sign in (30) is non-normalizable[11].

In our case notice that in the vicinity of the singularity the extra complex term to V_p is negligible implying that the solutions are expressed in terms of the two zero modes. The issue of the quantum mechanical behaviour of the singularity is now dependent on the norm of these eigenfunctions. We find that the norm of $\psi_2(\rho^*)$ is divergent. Therefore the Hamiltonian H is essentially self-adjoint.

Physically the naked singularity at the origin is a repulson leading a well-defined time evolution of scalar probes. This does not prevent the S-matrix from being non-unitary. Indeed let us consider the time evolution of a wave packet initially at infinity in the ρ^* direction. Noting that the eigenfunctions at infinity behave like free waves $e^{\pm i\omega \rho^*}$, we can expand this initial wave packet on the eigenvectors of H

$$\tilde{F}_0(\rho^*, 0) = \int d\omega a(\omega)(\psi_\omega^1(\rho^*) + i\psi_\omega^2(\rho^*)). \qquad (32)$$

The two eigenvectors behave like $\psi_\omega^1(\rho^*) = i\sqrt{\frac{2}{\pi|\omega|}}\cos(|\omega|\rho^* - \pi/4)$ and $\psi_\omega^2(\rho^*) = i\sqrt{\frac{2}{\pi|\omega|}}\sin(|\omega|\rho^* - \pi/4)$ at infinity. At any given time the wave packet becomes

$$\tilde{F}_0(\rho^*, t) = \int d\omega e^{i\omega t} a(\omega)(\psi_\omega^1(\rho^*) + i\psi_\omega^2(\rho^*)) \qquad (33)$$

evolving towards the singularity. Let us specialize to the neighbourhood of the singularity where all the generalized eigenfunctions are ω-independent

$$\tilde{F}_0(\rho^*, t) = (\int d\omega a(\omega)e^{iwt}) \, (\psi_1(\rho^*) + i\psi_2(\rho^*)). \qquad (34)$$

For a very large time corresponding to the very large initial value of ρ^* at the centre of the wave packet we find that the wave packet reaches the neighbourhood of the singularity. Moreover the wave packet becomes non-normalizable due to the $\psi_2(\rho^*)$ component. Therefore the infinite time limit of the time evolution operator is not unitary. This is due to the fact that the singularity is surrounded by an absorbing infinite well.

In a sense this provides a quantum mechanical resolution of the naked singularity. Indeed the classically attractive singularity is replaced quantum-mechanically by a finite absorbing region surrounding a repulson.

3 Graviton Absorption

3.1 Gravitons vs. Scalars

Let us analyse the absorption of gravitons polarized along the brane. This corresponds to perturbations of the metric of the brane by the incoming gravitons. In the Einstein frame the graviton equation reduces to the Laplace equation

$$\Delta h_{ij} = 0. \tag{35}$$

Consider waves of the form

$$h_{ij} = \epsilon_{ij} h e^{ik.x} \tag{36}$$

where ϵ is the polarization tensor and k is in the time direction. The polarization tensor must be traceless $\eta^{ij}\epsilon_{ij} = 0$ and transverse to k implying that $\epsilon_{0i} = 0$. Denoting by $\tilde{\epsilon}$ the spatial part of the polarization tensor we find a basis of these tensors given by off-diagonal symmetric matrices with $\tilde{\epsilon}_{ab} = \tilde{\epsilon}_{ba} = 1$ and zero otherwise along with diagonal matrices such that $\tilde{\epsilon}_{aa} = 1$, $\tilde{\epsilon}_{bb} = -1$, $a < b$. The latter are diagonal polarizations while the former are transverse polarizations. In the diagonal case put

$$h = g_{\rho\rho}^{1/2}\phi. \tag{37}$$

Then the scalar field ϕ satisfies the free wave equation

$$\nabla_\mu \nabla^\mu \phi = 0. \tag{38}$$

In the transverse case the function h satisfies the free scalar equation too.

We will calculate the absorption of gravitons by the non-BPS branes. It is convenient to define the conserved flux[17]

$$\mathcal{F}_{grav} = \frac{1}{2i} \int \sqrt{g} g^{\mu\nu} g^{\alpha\beta} g^{\rho\rho} (h_{\mu\alpha}^* \partial_\rho h_{\nu\beta} - \partial_\rho h_{\mu\alpha}^* h_{\nu\beta}). \tag{39}$$

The flux becomes

$$\mathcal{F}_{grav} = \frac{1}{2i} \int \sqrt{g} g^{ab} g^{cd} \tilde{\epsilon}_{ac} \tilde{\epsilon}_{bd} g^{\rho\rho} (h^* \partial_\rho h - \partial_\rho h^* h). \tag{40}$$

which is directly related to the scalar flux. In the following we shall compute the scalar absorption.

3.2 Non-BPS Graviton Absorption

The scattering potential requires the evaluation of $g^{\rho\rho}\sqrt{g} = \rho^{8-p}\tilde{f}_{++}$. This leads to the form factor

$$U = \rho^{8-p} + 2r_0^{7-p}\rho. \tag{41}$$

The potential follows after rescaling $z = \rho\omega$

$$V_p = -(1 + \frac{2\mu}{z^{7-p}})^{\alpha+\beta_+}(1 + \frac{\mu}{z^{7-p}})^{2(p-8)/(7-p)} + \frac{(8-p)(6-p)}{4z^2} - \mu^2\frac{(7-p)^2}{z^2(z^{7-p}+2\mu)^2} \tag{42}$$

where $\mu = (\omega r_0)^{7-p}$. For higher partial waves there is an extra repulsive contributions

$$\frac{l(l+7-p)}{\rho^2\tilde{f}_+^2}. \tag{43}$$

The potential has only a weak dependence on c_1.

Notice that the shape of the potential is different for $p = 6$ and $p < 6$. Let us first focus on the $p = 6$ case.

- **p=6**

We can distinguish three different regions. In the core for $z << \mu$ the potential reads

$$V_6 = -1 - \frac{1}{4z^2}. \tag{44}$$

This is the universal behaviour of the previous section which also appears for the naked singularities of cosmic string solutions[16]. Moreover the extra contribution from the higher partial waves is negligible compared to the $1/z^2$ behaviour.

The absorption cross section is obtained by considering incoming waves reaching the outer boundary of the core region $\rho \approx r_0$ and comparing this incoming flux to the one at infinity. To do so we need to study the potential outside the core

$$V_6 = -(1 + \frac{4k - 3c_1}{2}\frac{\mu}{z}) - \frac{\mu^2}{z^4}. \tag{45}$$

In the inner region $\mu << z << \mu^{1/3}$ for $\mu << 1$ the potential becomes

$$V_6 = -1 - \frac{\mu^2}{z^4}. \tag{46}$$

Finally in the outer region $z >> \mu^{1/3}$ the potential reads

$$V_6 = -1 - \frac{4k - 3c_1}{2}\frac{\mu}{z}. \tag{47}$$

In the outer region the solutions are known to be expressible in terms of confluent hypergeometric functions[17]

$$F_0 = \mathcal{A}_6 z e^{iz} \, {}_1F_1(1 - i\frac{4k - 3c_1}{4}\mu, 2; -2iz).$$ (48)

The amplitude $\mathcal{A}_6$ is obtained by matching with the inner region where the solution can be expressed in terms of Mathieu functions. It will be more transparent to use a duality transformation reducing the solutions to Hankel functions. Define

$$\tilde{z} = \frac{\mu}{z}$$ (49)

and

$$F_0(z) = \tilde{z}^{-1/2} \tilde{F}_0(\tilde{z}).$$ (50)

The equation in the dual variables is now

$$\frac{d^2\tilde{F}_0}{d\tilde{z}^2} + \tilde{z}\frac{d\tilde{F}_0}{d\tilde{z}} + (1 - \frac{1}{4\tilde{z}^2} + \frac{\mu^2}{\tilde{z}^4})\tilde{F}_0 = 0$$ (51)

which is a Bessel equation for $z << 1/2$. There is an overlapping interval for the solution of the dual equation and the solution in the outer region. More precisely the solution is expressed in terms of the Hankel function $H_{1/2}^{(1)}$ and reads

$$F_0 = i\sqrt{\frac{2}{\pi}\frac{z}{\mu}}e^{i\mu/z}$$ (52)

for $\mu << z << 1/2$. In this interval the arguments of the Hypergeometric and Hankel functions are small. The inner function behaves like $i\sqrt{\frac{2}{\pi}\frac{z}{\mu}}$ while the hypergeometric function matches this behaviour implying that

$$\mathcal{A}_6 = i\sqrt{\frac{2}{\pi}\frac{1}{\mu}}.$$ (53)

This specifies entirely the scalar wave function outside the core of the brane.

We can now evaluate the absorption cross section. To do so one needs to evaluate the ingoing fluxes at infinity and at the outer boundary of the core, i.e. $r = 2r_0$. The scattering cross section is given by

$$\sigma_6 = \frac{4\pi^2}{\Omega_2 w^2}\frac{\mathcal{F}_{2r_0}^{in}}{\mathcal{F}_\infty^{in}}$$ (54)

where $\Omega_d = 2\pi^{(d+1)/2}/\Gamma((d+1)/2)$.

Notice that the incoming wave for small μ behaves like $\mathcal{A}_6 e^{iz}$ at infinity. We obtain the flux per unit volume $\mathcal{F}_\infty^{in} = 4\pi\omega|\mathcal{A}_6|^2$. Similarly the flux at $2r_0$ is given by $\mathcal{F}_{2r_0}^{in} = 8/r_0$. Now the corresponding absorption cross section is given by

$$\sigma_6 = \frac{\pi r_0}{\omega}.$$ (55)

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The cross section diverges for small ω.

As already seen in [10] there is no decoupling limit of the brane modes from the bulk modes. In particular this can be shown by considering a non-BPS brane whose size scales with the string length and computing the absorption cross section in string units in the limit $l_s \to 0$. Dimensionally the radius r_0 is given by $r_0 \propto l_s$. In the small string length limit the ratio

$$\frac{\sigma_6}{l_s^2} \propto \frac{\pi}{l_s \omega} \tag{56}$$

measures the absorption in string units. As $l_s \omega << 1$ we find that the scalars and therefore the gravitons do not decouple from the brane.

- **p ≠ 6**

We can distinguish three different regions. In the core for $z << \mu^{1/(7-p)}$ the potential reads

$$V_p = -1 - \frac{1}{4z^2}. \tag{57}$$

This is the near singularity behaviour that has already been observed in the previous section. Moreover the extra contribution from the higher partial waves is negligible compared to the $1/z^2$ behaviour.

Let us now study the behaviour outside the core. In the inner region $\mu^{1/(7-p)} << z << \mu^{1/(9-p)}$ the potential reads

$$V_p = -1 + \frac{(8-p)(6-p)}{4z^2} - \mu^2 \frac{(7-p)^2}{z^{2(8-p)}}. \tag{58}$$

Finally in the outer region for $z >> \mu^{1/(9-p)}$ the potential decreases like

$$V_p = -1 + \frac{(8-p)(6-p)}{4z^2}. \tag{59}$$

On the last expression one can see that the potential decreases at infinity, as it increases around the origin there is a maximum for $z_* = O(\mu^{1/7-p})$ of height $V_p(z_*) = O(\mu^{-2/(7-p)})$. This is highly similar to the extremal case[17] where there is also a maximum for the potential when $p < 6$. The presence of a maximum in the potential should not be mistaken with a barrier preventing the penetration of the incoming waves. Indeed if one uses a more appropriate spherical representation of the Schrodinger equation one gets by putting $F_0 = z^{(8-p)/2} f_0$

$$-\frac{1}{z^{8-p}} \frac{d}{dz}(z^{8-p} \frac{df_0}{dz}) + \tilde{V}_p f_0 = 0 \tag{60}$$

where

$$\tilde{V}_p = V_p - \frac{(8-p)(6-p)}{4z^2}. \tag{61}$$

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The additional contribution cancels exactly the repulsive part of the potential leading to an attractive potential altogether.

Let us now concentrate on the absorption . To do so it is convenient to solve first the Schrodinger in dual variables

$$\tilde{z} = \frac{\mu}{z^{7-p}} \tag{62}$$

and

$$F_0(z) = \tilde{z}^{-1/2(7-p)} \tilde{F}_0(\tilde{z}). \tag{63}$$

The equation in the dual variables is now

$$\frac{d^2 \tilde{F}_0}{d\tilde{z}^2} + \tilde{z}\frac{d\tilde{F}_0}{d\tilde{z}} + (1 - \frac{1}{4\tilde{z}^2} + \frac{\mu^{2/(7-p)}}{(7-p)^2 \tilde{z}^{2+2/(7-p)}})\tilde{F}_0 = 0 \tag{64}$$

which is a Bessel equation for $z << (7-p)/2$. There is an overlapping interval for the validity of the solution of the dual equation and the solution in the outer region. More precisely the solution reads now

$$F_0 = i\sqrt{\frac{2}{\pi}} \frac{z^{(8-p)/2}}{\mu^{(8-p)/2(7-p)}} e^{i\mu/z^{7-p}} \tag{65}$$

for $\mu^{1/(7-p)} << z << (7-p)/2$. In the outer region the solution is a Bessel function of order $(7-p)/2$

$$F_0 = \mathcal{A}_p z^{1/2} J_{(7-p)/2}(z). \tag{66}$$

Now for small arguments $F_0 \sim \mathcal{A}_p \frac{z^{(8-p)/2}}{2^{(7-p)/2}\Gamma((9-p)/2)}$ implying that

$$\mathcal{A}_p = i\sqrt{\frac{2}{\pi}} 2^{(7-p)/2}\Gamma(\frac{9-p}{2})\mu^{(p-8)/2(7-p)}. \tag{67}$$

We can now deduce the flux at infinity

$$\mathcal{F}_\infty^{in} = \Omega_{8-p}\frac{\omega}{2\pi}|\mathcal{A}_p|^2. \tag{68}$$

Similarly the flux close the core is

$$\mathcal{F}_{r_*}^{in} = \Omega_{8-p}\frac{2(7-p)\omega}{\pi}\mu^{-1/(7-p)} \tag{69}$$

where $r_* = 2^{1/(7-p)}r_0$. This leads to the cross section

$$\sigma_p = \frac{(2\pi)^{8-p}}{\omega^{8-p}\Omega_{8-p}} \frac{\mathcal{F}_{r_*}^{in}}{\mathcal{F}_\infty^{in}} \tag{70}$$

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which reads

$$\sigma_p = \frac{(7-p)\Omega_{8-p}r_0^{7-p}}{\omega}.$$ (71)

The cross-section is divergent for soft probes.

Let us now consider a non-BPS brane with $r_0 \propto l_s$ and study the decoupling of gravity in the small string length regime. The absorption cross section behaves like

$$\frac{\sigma_p}{l_s^{8-p}} \propto \frac{(7-p)\Omega_{8-p}}{l_s\omega}$$ (72)

implying that the gravitons do not decouple from the inner region.

Another interesting point is the c_1 independence of σ_p. This entails that the probes are insensitive to the inner state of the brane, i.e. whether the tachyon possesses a vev.

Notice that we expect that higher curvature terms in the supergravity Lagrangian will modify the non-BPS solutions. Nevertheless the previous absorption cross section has been obtained from incoming waves penetrating the high curvature region where absorption takes place. Outside this region the non-BPS solutions should remain valid and therefore the absorption cross-section should keep the same features, i.e. the absence of decoupling of gravity.

4   Conclusions

We have studied the absorption of scalars and gravitons by non-BPS branes. One of the main features of non-BPS branes is the existence of a naked singularity. From the classical point of view this is an attractive singularity which needs to be resolved. Quantum mechanically when probed by gravitons and scalars the naked singularity appears to be a repulson reflecting incoming waves. Nevertheless the potential well around the repulson is so steep that wave-packets are trapped. This entails that the scattering S-matrix is not unitary. The absorption cross-section possesses an infra-red singularity which signals the non-decoupling of gravity.

In summary we have found that non-BPS branes are of a peculiar nature. Indeed the graviton probes do not reach the naked singularity while they remain strongly coupled to the neighbourhood of this singularity. Perhaps this is the supergravity way of observing that at short distances the predominant features of non-BPS branes spring from open string excitations. This suggests a possible resolution of the naked singularity by a non-supersymmetric *enhançon* mechanism[18, 19] whereby the $\rho \leq r_0$ region is smoothed out while the $\rho \geq r_0$ remains unchanged.

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